

PRICING EQUITY DEFAULT SWAPS

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ABSTRACT. Pricing credit-equity hybrids is a challenging task as the established pricing methodologies for equity options and credit derivatives are quite different. Equity default swaps provide an illuminating example of the clash of methodologies: from the equity derivatives viewpoint they are digital American puts with payments in installments and thus would naturally be priced by means of a local volatility model, but from the credit viewpoint they share features with credit default swaps and thus should be priced with a model allowing for jumps and possibly jump to default. The question arises of whether the two model classes can be consistent. In this paper we answer this question in the negative and find that market participants appear to be pricing equity default swaps by means of local volatility models not including jumps. We arrive at this conclusion by comparing a CEV model with an absorbing default barrier and a credit barrier model together with a credit-to-equity mapping that is calibrated to achieve consistency between equity option data, credit default swap spreads and historical credit transition probabilities and default frequencies.

1. INTRODUCTION

Equity default swap (EDS) contracts have recently been launched and are actively traded in increasing volumes. From the credit modelling viewpoint, EDS contracts are structured similarly to credit default swap (CDS) contracts, except that payouts occur when an *equity default* event happens, as opposed to a credit default event. Typical maturities are of 5 years and payment frequencies are semi-annual. By definition, equity default occurs whenever a given share price drops below 30% of the spot value at initiation of the contract. Equity default events for publicly traded companies are easily documentable.

EDS contracts command higher spreads than CDS contracts as credit defaults subsume equity defaults. This allows for several interesting trading strategies such as the EDS/CDS carry trade, whereby an investor sells EDS protection and buys CDS protection on the same name. In the case that no equity default event occurring prior to maturity, the investor will have gains due to the difference between EDS and CDS spreads. In the case that a credit default occurs the investor is hedged unless the recovery rate on the CDS leg is less than 50%. Losses occur if an equity default event occurs prior to maturity but is not accompanied by credit default. As we discuss in this article, model risk is also an important factor to take into consideration.

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Another typical trade is to enhance the yield of synthetic collateralized debt obligations (CDOs) by creating pockets of EDS positions or even creating a synthetic equity collateralized obligation (ECO) by a portfolio of only EDS contracts. The challenge from the pricing viewpoint is to determine the fair value for the ratio between CDS and EDS spreads and thus suitable replication strategies. In this paper, we find that this ratio is quite sensitive to the modeling assumptions.

From the equity modeling viewpoint, EDS contracts are structured as far out-of-the-money American digitals with payments in semiannual installments. The standard market practice for barrier options would be to use a local volatility model calibrated to European options. In the case of EDS contracts however, the practice is questionable due to the extreme out of the money character of the strike. In fact, the magnitude of the drop in share price needed to trigger the payout of an EDS will likely occur concurrently with a deterioration in the credit rating of the firm, implying major changes in the capital structure of the firm. In turn, this can lead to quite different values for implied volatilities and a modification of the corresponding implied volatility process.

The point of view we take in this paper is that in order to properly specify a model for EDS contracts one should calibrate to aggregate data for firms across all credit ratings to ensure that the model remains consistent if a drastic drop in equity price occurs.

Besides the volatility, a second issue is the presence of jumps. In credit models for credit default swaps, convertible bonds and credit baskets it is customary to include jumps. This is a common feature of all credit models which has substantial repercussions on the size of EDS spreads. Qualitatively, the presence of jumps increase the CDS spread to EDS spread ratio to approach 100%. In this paper we attempt to quantify the difference using the credit barrier models introduced in Albanese, Campolieti, Chen and Zavidonov (2003) and Albanese and Chen (2004), combined with a new technique for credit-equity mapping designed to price credit-equity hybrids.

In the version of Albanese et al. (2003), credit barrier models for the credit quality process are calibrated to market data in both the statistical and pricing measures. It was shown that under the statistical measure, historical averages of rating transition and default probabilities can be replicated with accuracy across all rating classes. In this analysis, we make use of through-the-cycle ratings, as opposed to point-in-time rating systems such as expected default frequencies. It is our opinion that cycle dependencies are an important but secondary effect which can be taken into account by a refinement of the model, but this would add technical complexities which there is merit to avoid in a first approximation. Upon applying a risk-neutralizing drift, the model spread rates in the pricing measure are in good agreement with market aggregate spread rates. The difference between model and market rates can be explained by financially meaningful forward liquidity spreads. In order to price EDS contracts, we use a credit quality to equity price mapping on top of the credit barrier model so that the information in the credit markets is imparted into the equity process.

Once mapped into an equity option model, we find that our calibrated credit barrier model leads to a local volatility profile quite similar to a CEV specification, except that jumps need to be present in the process to achieve consistency with historical transition probabilities. We thus compare EDS prices obtained with

two procedures: (i) by using the credit barrier model with jumps and a credit to equity map, and (ii) by calibrating a CEV local volatility pure diffusion model with absorption at zero to single name at-the-money implied equity option volatilities and CDS spreads. Using the CEV model, we find that the ratio of EDS to CDS spreads can be as large as 8:1. Instead, a credit barrier model including jumps can only be consistent with ratios of at most 2:1. Comparing with market quotes, we conclude that currently jumps do not appear to be priced.

The article is organized as follows: section 2 gives the specification of the credit to equity mapping and describes the calibration procedure. We then price EDS contracts in section 3. Section 4 prices EDS contracts using a pure diffusion process of CEV type with defaults. Section 5 concludes.

2. CREDIT QUALITY TO EQUITY MAPPING

As explained in the introduction, credit barrier models are calibrated to aggregate data, namely credit transition probabilities, historical default probabilities and average credit default swap spreads in the investment grade sectors. To reconcile the statistical with the pricing information, the market price of credit risk is estimated and yield spreads are disentangled into a sum of credit, taxation and liquidity spreads. Referring to our previous papers Albanese et al. (2003) and Albanese and Chen (2004) for a description of the mathematical framework and calibration issues, we describe here in detail the construction of the credit-to-equity mapping.

It is challenging to fit together in a comprehensive framework the three interacting shocks coming from the CDS market making, the stock market making and the volatility market making. There is no a-priori reason why these three shocks should be driven by a single factor. However, one would expect that conditioned to a rare event such as equity or credit default occurring, the three market makers would act the same way. Assuming conditional collinearity of the relevant shocks, one is led to construct one factor models where equity is deterministically linked to credit quality.

Define a credit quality variable y which can take values on a discrete lattice of points $\Lambda^N = \{y_n\}_{n=0}^N$ with time restricted to a discrete set of points $t_i = i\Delta t$ where Δt is a fixed time interval such as one quarter. The parameters for the specification of the pricing measure \mathcal{Q} are obtained by calibrating against aggregate corporate bond prices. The kernel of transition probabilities on the lattice Λ^N and on the discrete time lattice is denoted by:

$$U_{t_i, t_j}(m, n) = \mathcal{Q}(y(t_j) = y_n | y(t_i) = y_m) \quad 0 \leq i \leq j \leq I, \quad 0 \leq m, n \leq N.$$

In addition, we have an absorbing state y_{-1} which we consider to be the state of default and for which we have the probabilities of default:

$$U_{t_i, t_j}(m, -1) = \mathcal{Q}(y(t_j) = y_{-1} | y(t_i) = y_m) \quad 0 \leq i \leq j \leq I, \quad 0 \leq m \leq N.$$

Let $t_I = I\Delta t$ be the date up to which we need to generate the credit quality and equity processes. For a given debt-issuer which is also a publicly traded company, at time $t = 0$ we let the initial stock price be S_0 and the initial credit quality be $y(0)$. We wish to specify the equity process $S(t)$ as a function of the credit quality $y(t)$ for $t = t_1, \dots, t_I$.

Since credit quality is not a tradeable asset, the credit quality process need not be a martingale process even in the pricing measure. However, in order to avoid arbitrage opportunities the discounted equity process $e^{-(r-q)t}S(t)$ must be a

martingale process in the pricing measure, where r is the risk-free rate and q is the continuously compounded dividend yield, both of which we assume to be constant. Specifically, we require that:

$$(1) \quad \mathbf{E}^{\mathcal{Q}} \left[e^{-(r-q)t_j} S(t_i + t_j) | S(t_i) \right] = S(t_i)$$

where $0 \leq i \leq I$ and $0 \leq j \leq I - i$. To ensure that equation (1) is satisfied, we can specify the credit quality to equity mapping at maturity t_I by a univariate function Φ . For the equity prices at times t_i previous to maturity, we compound the expectation of Φ conditional to the credit quality at t_i . That is, at time $t = 0, t_1, \dots, t_I$, the stock price as a function of the credit quality is:

$$(2) \quad S(y(t), t) = e^{(r-q)t} S_0 \frac{\mathbf{E}^{\mathcal{Q}} [\Phi(y(t_I)) | y(t)]}{\mathbf{E}^{\mathcal{Q}} [\Phi(y(t_I)) | y(0)]}$$

The factor $\frac{S_0}{\mathbf{E}^{\mathcal{Q}} [\Phi(y(t_I)) | y(0)]}$ is included so that $S(y(0), 0)$ has the correct initial value.

Calibration. The credit to equity mapping for all times is characterized by the univariate function Φ . So that the default state y_{-1} of the credit quality variable corresponds to a zero stock price, the function Φ must satisfy $\Phi(y_{-1}) = 0$. From equation (2) we see that this ensures that $S(y_{-1}, t) = 0$ for all t .

Empirically, we would expect that in most instances an increase in credit quality will correspond to an increase in equity price. There are, however, exceptions to this general case. For example, if a firm issues additional shares, this has a detrimental effect on stock prices due to dilution, while cash reserves will increase and the credit quality will improve. The opposite happens in case of a share buy-back. We consciously ignore these effects and require that the equity price be a monotonic function of the credit quality at all times. This can be accomplished by restricting our choice of Φ to be monotonically increasing.

Having tried several alternatives, the method we settled on to specify Φ is to choose this function so that extreme moves in the equity correspond to a given drop in credit quality. Typically, EDS contracts pay out when the share price drops to 30% of its price when the contract is initiated. The approach we followed is to obtain statistics on the average rating change that occurs when drops of that magnitude occur in the associated equity, and then choose Φ to match the rating change for a representative firm with each initial rating. This method appears to be numerically robust and provides a reasonable fit also to at-the-money implied volatilities for equity options. Instead, the strategy of fitting only the latter implied volatilities alone proved to be more difficult to implement.

Φ is determined non-parametrically. The value of Φ is specified at a number of points on the lattice Λ^N and the values in between are determined using a spline. Specifically, for a set of integers k_1, \dots, k_M with $0 = k_1 < k_2 < \dots < k_M = N$ we specify the values of $\Phi(y_{k_1}), \dots, \Phi(y_{k_M})$. These values are specified so that 70% drops in equity price correspond to the desired rating change, for each initial rating.

We choose a set of L initial ratings with initial nodes $b_1, \dots, b_L \in \{0, \dots, N\}$. If the function Φ is specified, we can use equation (2) to find the function $S(y, t)$. From each initial node b_ℓ , we can find the closest node c_ℓ that corresponds to a 70% decrease in share price after t_I years as

$$|S(y_{c_\ell}, t_I) - 0.3S(y_{b_\ell}, 0)| = \min_{0 \leq i \leq N} |S(y_i, t_I) - 0.3S(y_{b_\ell}, 0)|$$

Initial Rating	Rating at 30%	Change
Caa	Caa	-0
B2	B3	-1
Ba2	B1	-2
Baa2	Ba2	-3
A2	Baa2	-3
Aa2	A2	-3
Aaa	A1	-4

TABLE 1. The target for calibrating to extreme price movements. The first column contains the seven initial ratings that we calibrate to. The second column contains the target rating which a 70% stock price fall corresponds to. The last column is the number of rating levels between the first and second columns.

We specify Φ as a linear spline in order to be able to apply a linear least squares routine. That is, we consider the values $\Phi(y_{k_1}), \dots, \Phi(y_{k_M})$ to be vertices of a piecewise linear function so that:

$$\Phi(y_j) = \theta + \sum_{m=1}^M [(\beta + \alpha_m)(y_{k_{m+1}} - y_{k_m})\mathbf{1}_{\{j \geq k_{m+1}\}} + (\beta + \alpha_m)(y_{k_j} - y_{k_m})\mathbf{1}_{\{k_m < j < k_{m+1}\}}]. \quad (3)$$

Here, $\theta > 0$ is the value of $\Phi(y_0)$, $\beta > 0$ is the minimum value that the slopes can have and $\beta + \alpha_m$ are the slope of Φ in the region $(y_{k_m}, y_{k_{m+1}})$, with $\alpha_m > 0$ for $m = 1, \dots, M$. Notice also that

$$\mathbf{E}^{\mathcal{Q}}[\Phi(y_{t_I}) | y(0) = y_i] = \sum_{j=0}^N U_{0,t_I}(i, j) \Phi(y_j). \quad (4)$$

Substituting equation (3) into (4), we see that the values $0.3S(y_{b_\ell}, 0)$ are linear functions of θ and the slopes α_m . Thus, if we specify targets for the c_ℓ , the problem is a linear least squares minimisation with non-negative constraints on the dependent variables θ and α_m , for $m = 1, \dots, M$.

As a simple example, we choose seven initial ratings: Aaa, Aa2, A2, Baa2, Ba2, B2, Caa. The rating migration that corresponds to a 70% drop in share price are shown in Table 1 and the values for the probability kernel U is taken from Albanese and Chen (2004). The Φ that leads to these rating migrations is shown in Figure 1. Notice the pronounced step-like behaviour of Φ . This can be seen as a reflection of the situation where the equity price declines sharply if the associated firm is downgraded by the rating agency.

It is instructive to see the local volatility function that is implied by this choice of Φ . In the discrete state case, the analog to the local volatility function is the

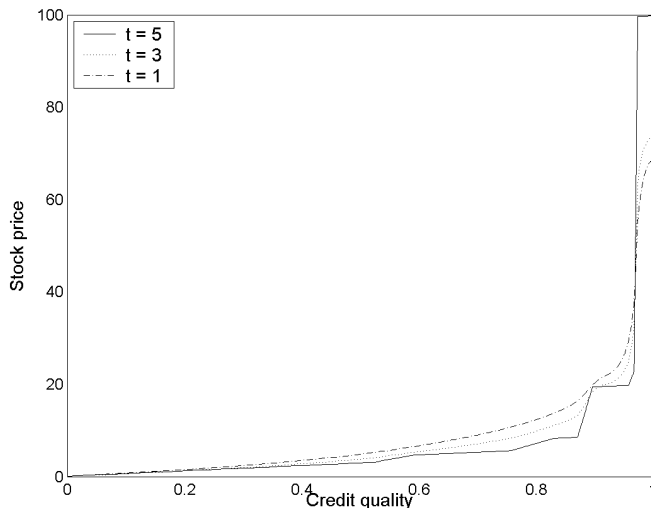


FIGURE 1. The function Φ labelled by $t = 5$ that gives rise to the rating migrations shown in Table 1. The lines labelled by $t = 3$ and $t = 1$ are $\mathbf{E}^{\mathcal{Q}}[\Phi(y(5)) | y(3)]$ and $\mathbf{E}^{\mathcal{Q}}[\Phi(y(5)) | y(1)]$, respectively. When the three are multiplied by $\frac{S_0}{\mathbf{E}^{\mathcal{Q}}[\Phi(y(5)) | y(0)]}$, we obtain the stock price as a function of credit quality for five, three and one years.

quantity:

$$(5) \quad \frac{\sqrt{\mathbf{E}^{\mathcal{Q}} \left[(S(y(t + \Delta t), t + \Delta t) - S(y(t), t))^2 | y(t) \right]}}{S(y(t), t) \Delta t},$$

which for a diffusion process is equal to the local volatility function in the limit $\Delta t \rightarrow 0$. The plot of equation (5) is given in Figure 2, when the function Φ is given by Figure 1. We see that it can be well approximated by the negative power function $\sigma(S) = \bar{\sigma} S^{-0.65}$. This property is far from obvious and suggests that a simple pure diffusion approximation to the equity process would correspond to a CEV specification of local volatility $\sigma(S) = \bar{\sigma} S^{-\beta}$.

3. PRICING AN EDS

We describe the pricing of a standard EDS. Our representative EDS has a notional amount of \$1 and requires semi-annual payments of $w_{\text{EDS}}/2$ until either the contract expires at maturity T , or there is a ‘equity default event’ of the stock price reaching 30% of the price when the EDS was struck. If an equity default event occurs, one last payment is made, equal to the accrued value of the payment from the last full payment. And, a payout is made to the protection buyer of an amount equal to one half the notional. The initial credit quality of the issuer of the equity is y_j .

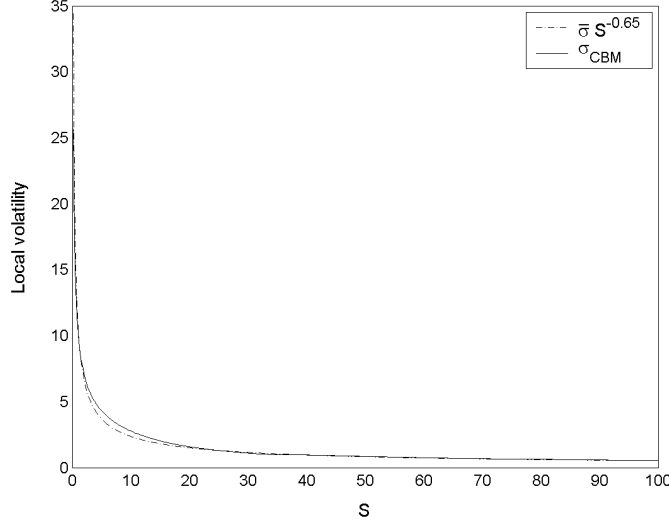


FIGURE 2.

Once we have specified a model for the stopping time corresponding to events of equity default, EDS contracts can be priced similarly to CDSs. The swap rate of an EDS is that rate for which the expected present value of payments is equal to the expected present value of payouts, so that the contract is at equilibrium. For ease of notation, we assume that payments occur with semi-annual frequency, and that the maturity T is an integer number of years. These two restrictions can be easily generalised.

Then, it can be shown in this discrete time setting that the swap rate s_{EDS} is:

$$s_{\text{EDS}} = \frac{\sum_{i=1}^I \hat{q}_{\text{CBM}}(t_i) e^{-rt_i}}{\left(1 - \sum_{i=1}^I \hat{q}_{\text{CBM}}(t_i)\right) \sum_{i=1}^{2T} e^{-ri/2} + \sum_{i=1}^I \left[\hat{q}_{\text{CBM}}(t_i) \left(\sum_{\ell=1}^{2t_i^*} e^{-r\ell/2} + 2(t_i - t_i^*) e^{-rt_i} \right) \right]} \quad (6)$$

where t_i^* is the last payment date before t_i , $\hat{q}_{\text{CBM}}(t_i)$ is the probability of equity default occurring at time t_i , with the dependency on the initial credit rating j suppressed. To calculate this probability, let $L = 0.3S_0$ be the barrier level at which the EDS pays out. Then, on the discretised time lattice $\{t_0 = 0, t_1, \dots, t_I = T\}$ and under the risk-neutral measure \mathcal{Q} , the probability of equity default occurring at time t_i is:

$$\hat{q}_{\text{CBM}}(t_i) = \sum_{\ell_i=-1}^{j_i-1} \sum_{\ell_{i-1}=j_{i-1}}^N \cdots \sum_{\ell_1=j_1}^N U_{t_0, t_1}(j, \ell_1) \cdots U_{t_{i-1}, t_i}(\ell_{i-1}, \ell_i)$$

where the j_i are such that

$$\begin{aligned} S(x_{j_{i-1}}, t_i) &\leq L \\ S(x_{j_i}, t_i) &> L \end{aligned}$$

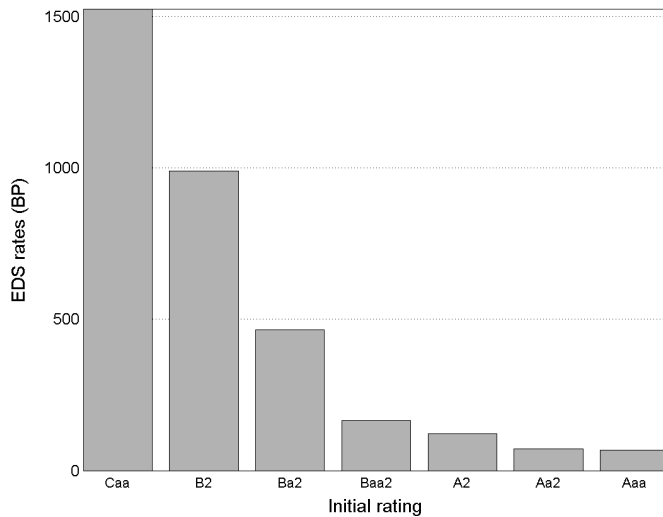


FIGURE 3. EDS rates for various initial credit ratings calculated using equation (6) and the function Φ shown in Figure 1

Numerical example. For the numerical example, we use a constant interest rate of 5% and a constant dividend yield of 3%. Using equation (6) with the function Φ shown in Figure 1 to specify the credit to equity mapping, the EDS rates calculated from the credit barrier model are shown in Figure 3. The CDS rates calculated from the credit barrier model as a percentage of the EDS rates are shown in Figure 4. We find that for high rated issues, the ratio of EDS to CDS spreads is about 2:1 while for low rated issues the same ratio is about 10:9. This compares to corresponding market ratios that typically range from 5:1 to 10:1, and can be as high as 20:1. See Figure 5, which plots the EDS spread against the CDS spread for firms in the DJ Europe Stoxx 50.

The reason why our EDS rates are so low relative to CDS rates is due to the presence of jumps in the credit barrier model. Since the default boundary is absorbing and jumps are added by subordinating on a gamma process, there is the possibility of jump to default.

For the sake of comparison, we also examine a pure diffusion equity model in which zero is an absorbing state. The local volatility shown in Figure 2 which is given by the credit barrier model and the credit to equity mapping specified by the Φ shown in Figure 1 can be approximated by $\bar{\sigma}S^{-0.65}$. This suggests that a pure diffusion approximation to the credit barrier model would be a stock price that follows a CEV process. In the next section, we elaborate on this model.

4. PRICING WITH A CEV MODEL WITH ABSORPTION

S_t is said to be a *constant elasticity of variance* (CEV) process if it solves the stochastic differential equation:

$$(7) \quad dS_t = \mu S_t dt + \bar{\sigma} S_t^{\beta+1} dW_t,$$

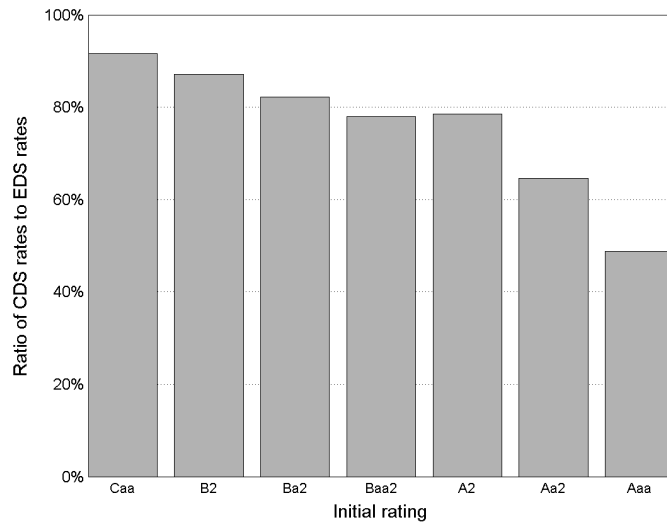


FIGURE 4. CDS rates as a percentage of EDS rates in Figure 3.

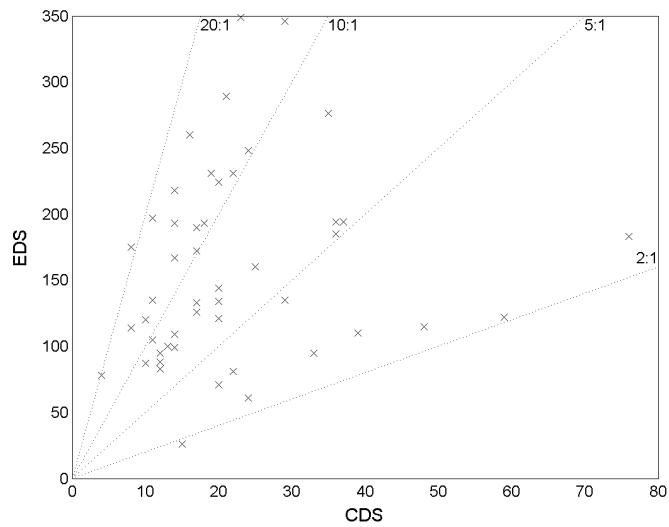


FIGURE 5. EDS spreads plotted against CDS spreads for firms in the Europe Stoxx 50 on August 4, 2004. The dotted lines are the lines of constant EDS to CDS spread ratio. Data courtesy of JP Morgan.

where W_t is standard Brownian motion, $\bar{\sigma} > 0$ and β is unrestricted. This process was originally introduced into the financial literature by Cox (1975). The geometric Brownian motion asset price process of Black and Scholes (1973) is obtained with the special case $\beta = 0$ and the square-root process of Cox and Ross (1976) is obtained with $\beta = -\frac{1}{2}$.

Under the risk-neutral measure \mathcal{Q}_{CEV} , we have $\mu = r - q$, where r is the risk-free rate and q the continuously compounded dividend yield.

The local volatility of the CEV process is $\sigma(S) = \bar{\sigma}S^\beta$. Due to the shape of the local volatility function for the credit barrier model with credit to equity mapping found in Figure 2, we focus on the case $\beta < 0$. Furthermore, from the boundary classification the CEV process has absorption at zero only for $\beta < 0$. Thus, the case $\beta \geq 0$ is not of interest for our purposes. Henceforth, all results and formulae will be presented for the case $\beta < 0$ only.

To calibrate the CEV equity process to price EDS contracts, we give formulae for the price of a European call and the CDS rate for an equity price that follows a CEV process. Thus, given an implied volatility (for say, a maturity of one year and at-the-money strike) and a CDS rate, we will be able to specify the parameters β and $\bar{\sigma}$ that reproduce these. With these, we can find the EDS rate.

Call price. Cox (1975) gives the European call price for a stock that follows a CEV process as:

$$C(S_0; K, T) = S_0 e^{-qT} Q(y(K, T); \eta, \zeta(S_0, T)) - e^{-rT} K (1 - Q(\zeta(S_0, T); \eta - 2, y(K, T))) \quad (8)$$

where T is the maturity, K is the strike, $Q(x; u, v)$ is the complementary noncentral chi-square distribution function with u degrees of freedom and non-centrality parameter v and

$$\begin{aligned} \eta &= 2 + \frac{1}{|\beta|}, \\ \zeta(S_0, T) &= \frac{2\mu S_0^{-2\beta}}{\bar{\sigma}^2 \beta (e^{2\mu\beta T} - 1)}, \\ y(K, T) &= \frac{2\mu K^{-2\beta}}{\bar{\sigma}^2 \beta (1 - e^{-2\mu\beta T})}. \end{aligned} \quad (9)$$

CDS rate. With the stock price following a CEV diffusion, the stopping time τ for credit defaults is

$$\tau = \inf(t > 0 : S_t = 0).$$

Thus, in order to find the CDS rate, we require the probability of absorption at zero of the stock price process.

The swap rate s_{CDS} of a CDS is

$$s_{CDS} = \frac{2 \int_0^T q_{CEV}(\tau) e^{-r\tau} \left[1 - \hat{R} - \hat{R}(\tau - \tau^*) C \right] d\tau}{(1 - Q_{CEV}(T)) \sum_{i=1}^{2T} e^{-ri/2} + \int_0^T q_{CEV}(\tau) \left[\sum_{i=1}^{2\tau^*} e^{-ri/2} + 2(\tau - \tau^*) e^{-r\tau} \right] d\tau} \quad (10)$$

where \hat{R} is the risk-neutral recovery rate and C is the coupon of the underlying bond as a fraction of the face, τ^* is the last CDS payment date before default occurring

at τ , $Q_{CEV}(t)$ is the probability of having defaulted by time t and $q_{CEV}(t)$ is the density of the probability of default. For the notation of both $Q_{CEV}(t)$ and $q_{CEV}(t)$, the dependency on the starting point S_0 has been suppressed.

The probability $Q_{CEV}(t)$ for a default occurring by time t is given in Cox (1975) as

$$\begin{aligned} Q_{CEV}(t) &= Q_{CEV}(S_t = 0 | S_0) \\ &= 1 - \Gamma\left(\nu, \frac{\zeta(S_0, t)}{2}\right), \end{aligned}$$

where ζ is given by (9), $\nu = \frac{1}{2|\beta|}$ and $\Gamma(\cdot, \cdot)$ is the incomplete gamma function. Then, the density $q_{CEV}(t)$ of the probability of default is just the derivative of $Q_{CEV}(t)$:

$$q_{CEV}(t) = \frac{\beta\mu\zeta(S_0, t)^\nu e^{-\zeta(S_0, t)/2}}{2^{\nu-1}\Gamma(\nu)(1 - e^{-2\mu\beta t})}.$$

Here, $\Gamma(\cdot)$ is the usual gamma function.

EDS rate. In analogy to the discrete time setting given in the previous section, in a continuous time model the swap rate s_{EDS} of an EDS is:

$$s_{EDS} = \frac{\int_0^T \hat{q}_{CEV}(\hat{\tau}) e^{-r\hat{\tau}} d\hat{\tau}}{\left(1 - \hat{Q}_{CEV}(T)\right) \sum_{i=1}^{2T} e^{-ri/2} + \int_0^T \hat{q}_{CEV}(\hat{\tau}) \left[\sum_{i=1}^{2\hat{\tau}^*} e^{-ri/2} + 2(\hat{\tau} - \hat{\tau}^*)e^{-r\hat{\tau}}\right] d\hat{\tau}} \quad (11)$$

where $\hat{\tau}^*$ is the last EDS payment date before equity default occurring at $\hat{\tau}$ with

$$\hat{\tau} = \inf(t > 0 : S_t \leq L),$$

where L is the level at which the payout of the EDS is triggered, typically 30% of the initial stock price. Also, $\hat{Q}_{CEV}(t)$ is the probability of equity default having occurred by time t and $\hat{q}_{CEV}(t)$ is the density of the probability of equity default. Again, for the notation of $\hat{Q}_{CEV}(t)$ and $\hat{q}_{CEV}(t)$ the dependency on the starting point S_0 is suppressed.

Thus, in order to compute the swap rate for an EDS of a stock following a CEV process using equation (11) we need to compute $\hat{Q}_{CEV}(t)$. Once this is obtained, $\hat{q}_{CEV}(t)$ is taken as the derivative of $\hat{Q}_{CEV}(t)$ and (11) can be computed. Now,

$$\begin{aligned} \hat{Q}_{CEV}(t) &= Q_{CEV}(\hat{\tau} < t | S_0) \\ &= \mathbf{E}[\mathbf{1}_{\{\hat{\tau} < t\}} | S_0] \end{aligned} \quad (12)$$

This probability is related to the rebate option given by Davydov and Linetsky (2003) and can be calculated as:

$$\mathbf{E}[\mathbf{1}_{\{\hat{\tau} < t\}} | S_0] = \frac{S_0^{\beta+\frac{1}{2}} e^{\epsilon x(S_0)}}{L^{\beta+\frac{1}{2}} e^{\epsilon x(L)}} \left[\frac{W_{\epsilon(1+\frac{1}{2\beta}), m}(x(S_0))}{W_{\epsilon(1+\frac{1}{2\beta}), m}(x(L))} - \sum_{n=1}^{\infty} \frac{2|\mu\beta| W_{k_n, m}(x(S_0)) e^{\lambda_n t}}{\lambda_n \left. \frac{\partial W_{k, m}(x(L))}{\partial k} \right|_{k=k_n}} \right]. \quad (13)$$

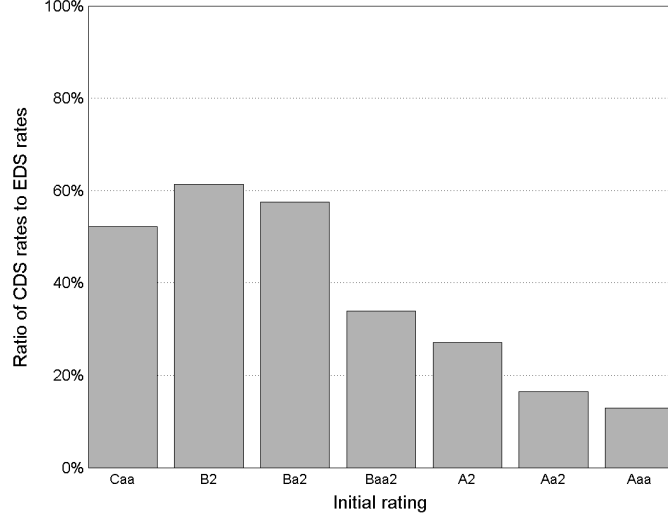


FIGURE 6. CDS rates as a percentage of EDS rates. The EDS rates were calculated using equation (13) for the CEV process.

Here,

$$(14) \quad \epsilon = -\frac{1}{2}\text{sign}(\mu)$$

$$(15) \quad m = \frac{-1}{4\beta}$$

$$(16) \quad x(S) = -\frac{|\mu|}{\bar{\sigma}^2\beta}S^{-2\beta}$$

and k_n is the n^{th} root of $W_{k,m}(x(L)) = 0$ and

$$(17) \quad \lambda_n = 2|\mu|\beta[k_n + \epsilon(2m - 1)]$$

In equation (13), the function $W_{k,m}(x)$ is the Whittaker function.

In practice, the roots k_n are computed numerically and the derivative terms in equation (13) are also computed numerically as there is no simple analytic formula for them. Then, using equation (13) for the function \hat{Q}_{CEV} , we can compute \hat{q}_{CEV} as the numerical derivative of \hat{Q}_{CEV} and both of these functions are used to calculate the EDS rate for a CEV process, using equation (11).

Again with an interest rate of 5% and a dividend yield of 3%, using β and $\bar{\sigma}$ such that the CDS rates match those from the credit barrier model and the one year at-the-money implied volatilities are an increasing function of the credit rating, we obtain Figure 6. We find that the ratios found using the CEV equity model are much closer to those found in market EDS and CDS spreads.

5. CONCLUSION

The ratio of EDS rates to CDS rates found in the CEV model vary from about 8:1 at the high ratings to about 2:1 in the low ratings. These are in the range of

current market ratios. On the other hand, the credit barrier model which includes jumps and is consistent with historical default and migration probabilities gives rise to ratios substantially closer to 1:1. We thus conclude that EDS contracts are currently being priced using pure diffusion processes with state dependent volatility and that the risk of credit jumps and jump to default is not priced.

A number of issues remain open though as our analysis is based on a number of assumption. The strongest ones are that there is only one factor driving both the credit and the equity process and we neglect information about economic cycles by calibrating to through-the-cycle rating transition probabilities. The extra randomness coming from the cycle indicators are captured in a stylized fashion by the jump model component. One could wonder whether a two factor model calibrated to point-in-time information would yield instead different results. Given that EDS contracts are typically struck with long maturities of about 5 years, intuition would suggest that cycle effect would not have a major impact on EDS/CDS ratios, although a more accurate analysis of this point would be useful.

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