King's College London

UNIVERSITY OF LONDON

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Candidate No: .... **Desk No:** ....

MSC EXAMINATION

CMFM03 FINANCIAL MARKETS (2010-2011)

SUMMER 2011

TIME ALLOWED: TWO HOURS

FULL MARKS WILL BE AWARDED FOR COMPLETE ANSWERS TO FOUR QUESTIONS. ONLY THE BEST FOUR QUESTIONS WILL COUNT TOWARDS GRADES A AND B, BUT CREDIT WILL BE GIVEN FOR ALL WORK DONE FOR LOWER GRADES.

WITHIN A GIVEN QUESTION, THE RELATIVE WEIGHTS OF THE DIFFERENT PARTS ARE INDICATED BY A PERCENTAGE FIGURE.

IMPORTANT, PLEASE READ: In the case of numerical answers, a concise numerical formula will suffice. For example, $x = \frac{1}{2}(3+8)$ instead of $x = 5.5$ will receive full marks.

NO CALCULATORS ARE PERMITTED.

TURN OVER WHEN INSTRUCTED

1. This is a question about interest rates, bond pricing and floating rate notes. Assume that the 6-month, 12-month, 18-month and 24-month zero rates are $r_0^{6m,6m} = 2\%$, $r_0^{1y,6m} = 2.25\%$, $r_0^{18m,6m} = 2.5\%$ and $r_0^{2y,6m} = 5\%$ respectively, with semiannual compounding.

(a) What is the one-year zero rate with (a) annual compounding and (b) quarterly compounding? [20%]

`Answer: From`

$$1 + r_0^{1y,1y} = \left(1 + \frac{1}{2} r_0^{1y,6m}\right)^2$$

`one finds`

$$r_0^{1y,1y} = r_0^{1y,6m} + \frac{1}{4}\left(r_0^{1y,6m}\right)^2.$$

`Also`

$$1 + r_0^{1y,1y} = \left(1 + \frac{1}{4} r_0^{1y,3m}\right)^4.$$

(b) What is the semiannually compounded forward rate for the 6-month period beginning in 12 months? [30%]

`Answer:`

$$1 + 0.5 \cdot f_{1y}^{6m,6m} = \frac{(1 + 0.5 \cdot r_0^{18m,6m})^3}{(1 + 0.5 \cdot r_0^{12m,6m})^2}.$$

(c) What is the value of the forward rate agreement (FRA) that promises to pay 6% (compounded semiannually) on a principal of \$5 million for the 6-month period starting in 18 months? [30%]

`Answer:`

$$1.06 \cdot 5M \cdot \left(1 + 0.5 r_0^{24m,6m}\right)^{-4} - 5M \cdot \left(1 + 0.5 r_0^{18m,6m}\right)^{-3}$$

(d) What equation solves the par yield with semiannual compounding of a 2-year bond paying semiannual coupons? [20%]

`Answer:`

$$\frac{0.5y}{1 + 0.5 r_0^{6m,6m}} + \frac{0.5y}{(1 + 0.5 r_0^{12m,6m})^2} + \frac{0.5y}{(1 + 0.5 r_0^{18m,6m})^3} + \frac{1 + 0.5y}{(1 + 0.5 r_0^{18m,6m})^4} = 1.$$

End of Question 1

2. These are questions about swaps. Assume that the term structure of continuously compounded zero rates for the USD and the GBP are both flat and are equal to $r_{USD} = 2\%$ and $r_{GBP} = 3\%$, respectively. The current value of the exchange rate is $X = 1.60\frac{GBP}{USD}$. Also assume that credit risk is negligible.

(a) A bank agreed to pay $r_s = 5\% USD$ per annum and to receive 3-month LIBOR USD in return on a notional principal of $10M USD with quarterly payments. The swap has a remaining life of 14 months. The 3-month LIBOR rate 1 month ago was 2.5%. What is the present value of the swap (from the bank's standpoint)? [30%]

`Answer:`

$$PV = 10M \cdot \left(e^{-\frac{2}{12}\cdot 0.02}(0.25 \cdot 0.025 + 1) - (1.05 \cdot e^{-\frac{14}{12}\cdot 0.02} + 0.05 \cdot e^{-\frac{2}{12}\cdot 0.02}) \right)$$

(b) Under the term of a swap agreement, a bank pays USD LIBOR semiannually on a nominal of 10M USD and receives GBP LIBOR semiannually on a nominal of 5M GBP. The swap will last 2 more years and at maturity nominals are exchanged. What is the present value in USD of this swap (from the bank's standpoint)? [35%]

`Answer:`

$$(5 \cdot 1.6 - 10) MUSD$$

(c) Consider a swap paying 3% in USD semiannually on a nominal of 15M USD and receiving GBP LIBOR quarterly on a nominal of 10M GBP. The swap matures in 2 years and at maturity nominals are exchanged. What is the PV in USD of this swap? [35%]

`Answer:`

$$\left(-15(0.03 \cdot e^{-\frac{1}{2}0.02} + 0.03 \cdot e^{-0.02} + 0.03 \cdot e^{-\frac{3}{2}0.02} + 1.03 \cdot e^{-0.04}) + 10 \cdot \right)) MUSD 1.6.$$

End of Question 2

3. This is a question about European options.

(a) Consider three call options of the same maturity and strikes K_1, K_2, K_3 respectively, such that $K_1 < K_2 < K_3$ and $K_2 - K_1 = K_3 - K_2$. Can one conceivably have a situation whereby the price of the option of strike K_2 is strictly higher than the average between the prices of the options of strikes K_1 and K_3? [25%]

`Answer:`

`Such a situation could conceivably manifest itself only if there was a value of the stock price` S `such that`

$$(S - K_2)_+ > \frac{1}{2}(S - K_1)_+ + \frac{1}{2}(S - K_3)_+.$$

`But there is no such a value of` S`. Equality is reached for` $S \geq K_3$ `but the strict inequality is not verified for any value of` S`.`

(b) Consider three put options of the same maturity and strikes K_1, K_2, K_3 respectively, such that $K_1 < K_2 < K_3$ and $K_2 - K_1 = K_3 - K_2$. Can one conceivably have a situation whereby the price of the option of strike K_2 is strictly higher than the average between the prices of the options of strikes K_1 and K_3? [25%]

`Answer:`

`Such a situation could conceivably manifest itself only if there was a value of the stock price` S `such that`

$$(K_2 - S)_+ > \frac{1}{2}(K_1 - S)_+ + \frac{1}{2}(K_3 - S)_+.$$

`But there is no such a value of` S`. Equality is reached for` $S \leq K_1$ `but the strict inequality is not verified for any value of` S`.`

(c) The price of a European call maturing in 6 months with strike \$30 is \$2. The underlying stock price is \$29. A dividend of \$0.50 is expected in 2 months and again in 5 months. Assume also that continuously compounded interest rates are constant at 2%. What is the price of a European put maturing in 6 months struck at \$30? [25%]

`Answer:`

`Since`

$$(S-K)_+ - (K-S)_+ = S-K$$

`the price of the put is`

$$2 - 29 + 0.5e^{-\frac{1}{6}0.02} + 0.5e^{\frac{-5}{12}0.02} + 30e^{-\frac{1}{2}0.02}.$$

(d) Consider a European option on the underlying S of maturity 1 year and payoff $S^2/100$. Suppose the spot price is $S_0 = \$10$. Suggest an approximate replication strategy for this derivative by means of positions in a bond maturing in one year, the stock itself and several call options. Can one make use of put options instead of call options for replication purposes? [25%]

`Answer: One has to implement a polygonal approximation of the parabola. Put options can also be used.`

End of Question 3

4. This is a question about equity options in the Black-Scholes model. Consider a stock whose spot price is $S_0 = \$10$ and whose volatility is $\sigma = 20\%$. Also assume that the short rate is constant at $r = 3\%$.

(a) Value a European option with payoff $S^2/100$ and maturity in one year under the Black-Scholes model. [50%]

`Answer:`

`In the Black-Scholes model`

$$S_t = S_0 e^{\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

`Hence the value of the option is`

$$V = e^{-rt} E_0 \left[e^{2\left(r - \frac{\sigma^2}{2}\right)t + 2\sigma W_t} \right] = e^{(r-\sigma^2)t}.$$

(b) Consider a European option with payoff $\left(\frac{S}{10}\right)^\alpha$ and maturity in one year. Compute the parameter α so that this option is worth \$1. [50%]

`The value of the option is`

$$V = e^{-rt} E_0 \left[e^{\alpha\left(r - \frac{\sigma^2}{2}\right)t + \alpha\sigma W_t} \right] = e^{((\alpha-1)r + (\alpha^2-\alpha)\frac{\sigma^2}{2})t}.$$

`Hence:` $\alpha = -\frac{2r}{\sigma^2}$.

End of Question 4

5. This is a miscellaneous question.

(a) Explain under what circumstances a bank is subject to credit risk when it negotiates two offsetting interest rate swaps. [20%]

`Answer: The bank is subject to credit risk when the counterparties for the two swaps are different.`

(b) Assuming that there is no counterparty credit risk, explain why a physically settled swaption can be regarded as an option on a fixed rate annuity. [20%]

`Answer: At maturity a physically settled swaption delivers a swap at pre-agreed swap rate. Since equilibrium swaps have zero value, in case there is no counterparty credit risk it is legitimate to subtract an equilibrium swap from the deliverable. The difference is a fixed rate annuity.`

(c) With the same settings as in (b) but in case there is counterparty credit risk, explain whether the situation changes and in what ways. [20%]

`If there is counterparty credit risk, introducing a swap at equilibrium in the portfolio would change the risk profile. One cannot thus introduce such a fictitious position as an accounting expedient to reduce a swaption to an option on an annuity.`

(d) Derive a put-call parity relation for European swaptions. [20%]

`Answer: The difference between a payer swaption and a receiver swaption with the same maturity is equivalent to a forward starting swap, starting at the swaptions maturity and with rate equal to the strike.`

(e) A company has an issue of executive stock warrants outstanding. Should dilution be taken into account in the valuation of these options? Explain. [20%]

`Answer:`

`Yes it should as dilution results automatically upon exercise and leads to stock depreciation.`

End of Question 5