King's College London

UNIVERSITY OF LONDON

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Candidate No: .... **Desk No:** ....

MSC EXAMINATION

CMFM03 - 2010 WARMUP FINANCIAL MARKETS

SUMMER 2010

TIME ALLOWED: TWO HOURS

FULL MARKS WILL BE AWARDED FOR COMPLETE ANSWERS TO FOUR QUESTIONS. ONLY THE BEST FOUR QUESTIONS WILL COUNT TOWARDS GRADES A AND B, BUT CREDIT WILL BE GIVEN FOR ALL WORK DONE FOR LOWER GRADES.

WITHIN A GIVEN QUESTION, THE RELATIVE WEIGHTS OF THE DIFFERENT PARTS ARE INDICATED BY A PERCENTAGE FIGURE.

IMPORTANT, PLEASE READ: In the case of numerical answers, a concise numerical formula will suffice. For example, $x = \frac{1}{2}(3+8)$ instead of $x = 5.5$ will receive full marks.

NO CALCULATORS ARE PERMITTED.

TURN OVER WHEN INSTRUCTED

1. This is a question about interest rates, bond pricing, and bond futures.

(a) Discount bonds of maturities 6, 12, 18 and 24 months are $Z(6m) = 0.9990$, $Z(1y) = 0.9970$, $Z(18m) = 0.9940$ and $Z(2y) = 0.9900$. How would you estimate the short rate in basis points? What is the forward rate of the 18-months-into-6-months year zero coupon bond? [20%]

The short rate is approximately 20 bp.

The forward price of a 6m-into-one year zero coupon bond is

$$F(18m, 6m) = \frac{Z(2y)}{Z(18m)} \approx 0.9960. \tag{1}$$

The forward rate $f(18m, 6m)$ is such that

$$F(18m, 6m) = \frac{1}{1 + 0.5f(18m, 6m)} \tag{2}$$

and hence $f(18m, 6m) \approx 80bp$

(b) Consider a two-year swap with semiannual frequency on both legs and the first coupon payment in 6 months. [45%]

(i) What is the equilibrium swap rate?

The equilibrium swap rate is

$$r_s = \frac{1 - Z(2y)}{Z(6m) + Z(1y) + Z(18m) + Z(2y)}. \tag{3}$$

(ii) Assuming the swap is at equilibrium, what is the yield to maturity of the fixed leg?

The yield to maturity Y for the fixed leg of the swap solves the equation:

$$1 = PV(Y) \tag{4}$$

where

$$PV(Y) = r_s \sum_{m=1,..4} \frac{1}{(1 + 0.5Y)^m}. \tag{5}$$

(iii) If the swap rate is equal to the equilibrium swap rate plus 10bp, how is the PV of the swap affected assuming that the holder receives fixed and pays float??

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The PV of the swap increases by

$$0.001 * (Z(6m) + Z(1y) + Z(18m) + Z(2y)). \tag{6}$$

(c) Consider a forward contract delivering a 18m swap in 6 m. Determine the swap rate in such a way that the forward contract is at equilibrium. [35%]

The equilibrium swap rate is

$$r_s = \frac{Z(6m) - Z(2y)}{Z(1y) + Z(18m) + Z(2y)}. \tag{7}$$

End of Question 1

2. This is a question about variance swaps and trading strategies in futures.

The payoff of a corridor variance swap is defined as $RV - SR^2$ where SR is a constant expressed in units of volatility and RV is the realized variance of arithmetic returns given by:

$$RV = \frac{1}{N} \sum_{i=0}^{N-1} 1(L < S_{t_i} < H) \left(\frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}} \right)^2. \tag{8}$$

Here $L < H$ are the lower and upper barriers of a corridor for stock prices and $1(L < S_{t_i} < H)$ is the characteristic function for the corridor, equal to 1 if $L < S_{t_i} < H$ and to 0 otherwise.

(a) Derive a power series approximation truncated to the second order for:

$$1(L < S_{t_i} < H) \log \frac{S_{t_{i+1}}}{S_{t_i}}. \tag{9}$$

[10%]

$$1(L < S_{t_i} < H) \log \frac{S_{t_{i+1}}}{S_{t_i}} = 1(L < S_{t_i} < H) \log \left(\frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}} + 1 \right) \approx 1(L < S_{t_i} < H) \frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}} \tag{10}$$

(b) Based on the Taylor expansion for log-returns in the corridor and assuming that interest rates are zero, show that realized variance can be expressed as follows:

$$RV \approx \frac{1}{N} \sum_{i=0}^{N-1} 1(L < S_{t_i} < H) \left(\frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}} \right)^2 \approx -\frac{2}{N} 1(L < S_{t_i} < H) \sum_{i=0}^{N-1} \log \frac{S_{t_{i+1}}}{S_{t_i}} + \frac{2}{N} 1(L < S \tag{11}$$

[10%]

`It suffices to sum up and equate both sides in the Taylor expansion above in (10), i.e.`

$$\frac{1}{N} \sum_{i=0}^{N-1} 1(L < S_{t_i} < H) \log \frac{S_{t_{i+1}}}{S_{t_i}} \approx \frac{1}{N} \sum_{i=0}^{N-1} 1(L < S_{t_i} < H) \frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}} - \frac{1}{2} \frac{1}{N} \sum_{i=0}^{N-1} 1(L < S_{t_i} < H \tag{12}$$

(c) Assume interest rates are zero. Explain how the sum

$$\frac{2}{N}\sum_{i=0}^{N-1} 1(L < S_{t_i} < H)\frac{S_{t_{i+1}} - S_{t_i}}{S_{t_i}}. \tag{13}$$

can be obtained by means of a strategy in futures. What is the cost of this strategy? [40%]

`In case interest rates are zero, the strategy in futures consists in taking a position in the stock in` $\frac{1}{S_{t_i}}$ `on the` $i-$`th day and holding it for one day only. The position is taken only if the initial stock price is within the corridor, while otherwise no action is taken. If interest rates are zero, the futures price over one day is equal to the stock spot price and hence the cost of this strategy is zero.`

(d) How does one replicate the final log-payoff with a series of positions in calls and puts? [40%]

End of Question 2

3. This is a question about interest rate swaps and floating rate notes.

(a) [25%] Consider a 10 year floating rate not with semi-annual coupons. Give a precise definition of the coupon amounts of an FRN with payments in advance, i.e. a contract that (i) can be replicated without model risk, (ii) its coupon amounts depend only on rates prevailing at the coupon date, (iii) the nominal amount is paid on the last coupon date and (iv) the note trades at par at inception.

(b) Consider a 10 year FRN with payments as in the previous question one day before the sixth coupon payment and one day after the sixth coupon payment. What is the PV on these dates? [25%]

(c) Consider a floating rate annuity defined as a floating rate note but without the nominal payment at the final maturity? [25%]

`The price of the floating rate note at a cash flow date is not subject to interest rate risk. The price in between two cash flow dates depends on the rate corresponding to the maturity given by the next cash flow date.`

(d) Consider a forward starting swap, with start date in 2 years and tenor 10 years. Give a formula for the equilibrium swap rate at current time and provide a derivation. [25%]

`Assuming to be specific that the coupon frequency is semiannual, the formula is`

$$SR_t = \frac{Z_t(2y) - Z_t(10y)}{\sum_{j=5}^{20} \tau Z_t(T_j)}. \tag{14}$$

End of Question 3

4. This is a question about equity options.

(a) Assuming no dividends and that the stock cannot default, can one replicate a forward contract at a given maturity by means of a position in a call and a position in a put options? How does the answer change if the stock pays dividends? How does the answer change if one allows for the possibility that the worth of the stock price vanishes prior to maturity due to default? [30%]

`In all situations, a forward contract is replicated by a long position in a call struck at the forward price combined with a short position in a put struck at the forward price. The possibility of dividends and default events do not affect the conclusion.`

(b) Explain how to replicate a futures contract on a non-dividend paying stock. What is the cost of replication? How does the answer change if the stock pays dividends? [30%]

`To replicate a long position in a futures contract on a non-dividend paying stock one can buy the stock funding the purchase with a stream of one-day loans, while depositing in a cash account the difference between the stock price realized on each day and the corresponding futures price on the previous day.`

(c) Consider the capped logarithmic payoff of European type

$$\min\left(-\log\left(\frac{S_T}{S_0}\right), \mathrm{Cap}\right) \tag{15}$$

where T is the maturity. Explain how to replicate this payoff approximately with a strategy involving only out of the money call options, out of the money put options, the stock and cash. [40%]

`To replicate this payoff, one would hold an amount equal to the cap in cash, a short position in the stock of size`

$$-\frac{1}{S_0}, \tag{16}$$

`a short position in a put struck at` $S_0 e^{-\mathrm{Cap}}$ `and a continuum of long positions in call and put options at strikes` $K > S_0 e^{-\mathrm{Cap}}$ `of (infinitesimal) position` $n(K)dK$ `to match the convexity of the log payoff, i.e. such that`

$$n(K) = \frac{1}{K^2}. \tag{17}$$

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End of Question 4

5. This is a question about forward and futures contracts.

(a) A short forward contract on gold that was negotiated three months ago will expire in one year and has delivery price of \$980 per ounce. The current price of gold is \$950 per ounce, the annual storage cost is \$0.50 per ounce and the risk-free rate is 3% per year (simply compounded).

(i) Calculate the current equilibrium forward price of gold for delivery in one year.

`The equilibrium forward price is`

$$(\$950 + \$0.50)(1 + 3\%) \approx 979 \tag{18}$$

(ii) Calculate the current value of the forward contract issued three months ago. [25%]

`The value is` $\frac{\$1}{1+3\%}$.

(b) On December 3rd 2008, at the Comex in New York, December gold futures were in backwardation. December 31 deliveries were quoted at 2% discount to spot, while gold futures with delivery February 27 were quoted at 0.29% discount to spot. (All percentages annualized.) How would you interpret these quotes? [25%]

`Since carry costs are positive and interest rates are also positive, the remaining factor that can potentially account for backwardation is the credit risk of the short party promising delivery.`

(c) A bank sells forward contracts on oil with delivery in one year to two different counter-parties. The contracts are otherwise identical but the forward price to one counter-party is \$45 per barrel, while the other counterparty pays \$45.5 per barrel. What reasons can motivate the difference? By how much would the futures prices on the same underlying differ among the two counter-parties? [25%]

`The two counter-parties may have different credit risk and/or there could be a different correlation between the probability of counter-party default and a rally in oil prices.`

(d) Metallgesellshaft used to be a major firm that would sell forward contracts for oil delivery to clients hedging them with futures contracts. In December 1993 the strategy gave rise to mark-to-market losses exceeding 1.5 billions. How could this hedging strategy go so wrong? [25%]

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The hedging strategy could go wrong because the futures positions gives rise to a continuous stream of cash flows. To cover, Metall-Gesellshaf had to take long futures position in the underlying. Since oil prices fell in that period, there were a series of margin calls which deteriorated the cash position of the firm and affected its credit rating, raising as a consequence its financing costs.

End of Question 5