King's College London

University Of London

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ATTACH this paper to your script **USING THE STRING PROVIDED**

Candidate No: .... **Desk No:** ....

MSc Examination

CMFM03 Financial Markets (2009-2010)

Summer 2010

Time Allowed: Two Hours

Full marks will be awarded for complete answers to FOUR questions. Only the best FOUR questions will count towards grades A and B, but credit will be given for all work done for lower grades.

Within a given question, the relative weights of the different parts are indicated by a percentage figure.

IMPORTANT, PLEASE READ: In the case of numerical answers, a concise numerical formula will suffice. For example, $x = \frac{1}{2}(3+8)$ instead of $x = 5.5$ will receive full marks.

NO calculators are permitted.

TURN OVER WHEN INSTRUCTED

1. This is a question about interest rates, bond pricing and floating rate notes. Assume that the discount bonds of maturities 6, 12, 18 and 24 months for the GBP are worth $Z(6m) = 0.9933$, $Z(1y) = 0.9863$, $Z(18m) = 0.9761$ and $Z(2y) = 0.9633$.

(a) What is the annually compounded forward rate of the 1-year-into-1-year year zero coupon bond? [20%]

`The forward price of a 1y-into-1y zero coupon bond is`

$$F(1y, 2y) = \frac{Z(2y)}{Z(1y)} \approx 0.9767. \tag{1}$$

`The annually compounded forward rate` $f(1y, 2y)$ `is such that`

$$F(1y, 2y) = \frac{1}{1 + f(1y, 2y)}. \tag{2}$$

`Approximately` $f(1y, 2y) \approx 0.0239bp$

(b) Consider a two-year floating rate note with nominal $N = 1MGBP$, semi-annual frequency on both legs, payments in arrears equal to the 6m rate plus 50 bp, the first coupon payment in 6 months and final repayment of the nominal $N = 1MGBP$ at maturity. What is the present value of the floating rate note? [30%]

`The present value of the floating rate note is`

$$PV = N + 0.0050 \cdot N \cdot (Z(6m) + Z(1y) + Z(18m) + Z(2y)). \tag{3}$$

(c) Consider a two-year floating rate annuity with nominal $N = 1MGBP$, semiannual frequency on both legs, payments in arrears equal to three times the 6m rate, the first coupon payment in 6 months and no nominal repayment at maturity. What is the present value of the floating rate annuity? [30%]

`The present value of the floating rate annuity is`

$$PV = 3N \cdot (1 - Z(2y)). \tag{4}$$

(d) Consider a forward contract delivering a 1 year swap with semiannual payments in 1 year. Determine the equilibrium forward swap rate. [20%]

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The equilibrium forward swap rate is
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$$r_s = \frac{Z(1y) - Z(2y)}{0.5Z(18m) + 0.5Z(2y)}. \tag{5}$$

End of Question 1

2. This is a question about normal variance swaps on a stock. Let S_i be the stock price i days after the issue date. If the maturity is in N days, the payoff of a normal variance swap is $RV_N - SR$, where the normal realized variance is defined as follows:

$$RV_N = \frac{1}{N}\sum_{i=1}^{N}(S_i - S_{i-1})^2. \tag{6}$$

(a) Derive the following formula [30%]

$$\sum_{i=1}^{N}(S_i - S_{i-1})^2 = S_N^2 - S_0^2 - 2\sum_{i=1}^{N}(S_i - S_{i-1})S_{i-1}. \tag{7}$$

`Answer:`

$$\sum_{i=1}^{N}(S_i - S_{i-1})^2 = \sum_{i=1}^{N} S_i^2 + S_{i-1} - 2S_iS_{i-1} \tag{8}$$

$$= S_N^2 - S_0^2 - 2\sum_{i}(S_i - S_{i-1})S_{i-1}. \tag{9}$$

(b) In case interest rates are zero, explain how the sum

$$-\frac{2}{N}\sum_{i=1}^{N}(S_i - S_{i-1})S_i. \tag{10}$$

can be obtained by means of a strategy in futures. What is the cost of this strategy? [35%]

`In case interest rates are zero, the strategy in futures is costless and consists in taking a position in` $-\frac{2}{N}S_i$ `contracts on the` $i-$`th day.`

(c) How might one approximately replicate the payoff

$$\frac{1}{N}(S_N^2 - S_0^2) \tag{11}$$

with a series of positions in the stock, a discount bond maturing on the same date as the variance swap, out of the money calls and out of the money puts? Compute exactly the position in the stock and in the discount bond and indicate an algorithm to determine the positions in out of the money puts and calls. Are the calls long or short? Are the puts long or short? [35%]

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One enters in positions in $\frac{2S_0}{N}$ shares to match the payoff derivative at S_0, in a position of $-\frac{2S_0^2}{N}Z_0(T)$ in the bond maturing at expiry T. The positions on puts are long and the positions in calls are long. The position amounts are computed in such a way to best approximate the given function by a polygonal payoff.

End of Question 2

3. This is a question about ISDA standard USD swaps. These are interest rate swaps where party A pays to party B fixed coupons proportional to the swap rate on a semiannual basis and receives LIBOR on a quarterly basis.

(a) Consider a 20 year USD interest rate swap with payments in arrears. Give a formula for the floating coupon amount defined in such a way that the swap present value can be valued uniquely off the discount curve. [20%]

`The floating coupon amount on the cash flow date` T_i `for the floating leg is equal to` $NL_{T_i-3m}(T_i)$`, where` $L_{T_i-3m}(T_i)$ `is the quarterly compounded LIBOR rate prevailing 3 months before the given cash flow date` T_i `and maturing at time` T_i`.`

(b) Suppose that the swap in (a) receives payments in advance. Now give a formula for the floating coupon. [20%]

`The floating coupon amount on the cash flow date` T_i `for the floating leg is equal to` $\frac{NL_{T_i}(T_i)}{1+0.25\cdot L_{T_i}(T_i)}$`, where` $L_{T_i}(T_i)$ `is the quarterly compounded LIBOR rate prevailing at the given cash flow date` T_i`.`

(c) Find an expression for the equilibrium swap rate for a USD swap in arrears at contract inception. [20%]

`The equilibrium swap rate is`

$$SR_0 = \frac{1 - Z(T_N)}{\sum_j 0.5 Z(\bar{T}_j)} \tag{12}$$

`where the` $\bar{T}_j$ `are the cash flow dates on the fixed leg.`

(d) Consider a variation on a standard USD swap where party A pays to party B fixed coupons and receives a quarterly coupon equal to the semi-annual LIBOR rate that prevailed 6 months prior to the coupon date. The latter LIBOR rates are computed using the semi-annual compounding rule over semi-annual periods. Assume that the first coupon payment on the floating leg occurs after 6 months and the subsequent ones happen at quarterly frequency.

Can this swap be uniquely valued off the discount curve? If the answer is affirmative, provide a formula. If the answer is negative, state what additional information is required. [40%]

`The exotic swap can still be valued off the discount curve as the floating coupon payments can be generated by a strategy whereby one combines a replication strategy starting with half the nominal` at the issue date and iterates semiannual deposits with a second strategy starting half the nominal one quarter after the issue date and iterates. Assuming that the tenor entails an odd number of quarter, we have that

$$SR_0 = 0.5 * \frac{1 - Z(T_N - 3m) + Z(3m) - Z(T_N)}{\sum_j 0.5 Z(\bar{T}_j)} \tag{13}$$

If the tenor instead entails an even number of quarter, we have that

$$SR_0 = 0.5 * \frac{1 - Z(T_N) + Z(3m) - Z(T_N - 3m)}{\sum_j 0.5 Z(\bar{T}_j)} \tag{14}$$

End of Question 3

4. This is a question about foreign exchange options.

(a) Consider a situation where the domestic currency is the JPY and the foreign currency is the USD. Suppose that the spot rate is 85 JPY = 1 USD, that the 1 year rate on the USD is 120 bp and the 1 year rate on the JPY is 20 bp. What is the forward price to purchase 1M yen in one year? How can one replicate the forward contract? [30%]

The forward price is

$$F = \frac{Z^f(1y)}{Z^d(1y)} \cdot 85 = \frac{1 + 0.002}{1 + 0.012} \cdot 85 \tag{15}$$

where $Z^f(T)$ is the foreign discount factor and $Z^d(T)$ is the domestic discount factor of maturity T. A forward contract is replicated by buying the $\frac{1M}{1+0.002}$ yen, funding the purchase by means of a 1-year loan in dollars and depositing the acquired currency in a one-year deposit.

(b) Consider two currencies, USD and JPY and an instrument of maturity 10 years and two nominals N_1 in dollars and N_2 in yen. Let us assume that $N_1 = 1M$ dollars and that $N_2 = 85M$ yen. The instrument is such that the holder receives USD Libor in arrears semiannually in USD on the nominal N_1 and pays semiannual JPY Libor in arrears on the nominal N_2 in yen. The deal also entails a final cash exchange. Determine the final cash exchange in such a way that the value of the deal at inception is zero. [40%]

The amount to be received is N_2 in yen at expiry minus N_1 in dollars at expiry.

(c) Consider the following payoff of European type

$$\max(0, 20 - |X_T - 85|) \tag{16}$$

where T is the maturity, and X_T is the USD/JPY exchange rate at time T. Explain how to replicate this payoff with a strategy involving only call options. [30%]

The nominal in JPY is equal to the forward price in JPY to purchase 1M USD in one year time, i.e.

$$\max(0, 20 - |X_T - 85|) = \max(X_T - 65, 0) - 2\max(X_T - 85, 0) + \max(X_T - 105, 0). \tag{17}$$

End of Question 4

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5. This is a miscellaneous question.

(a) A forward contract on oil that was negotiated three months ago will expire in one year and has delivery price of \$70 per barrel. The current price of oil is \$85 per barrel, the annual storage cost is \$5.0 per barrel and the risk-free rate is 3% per year (simply compounded).

(i) Calculate the current equilibrium forward price of oil for delivery in one year. [15%]

`The equilibrium forward price is`

$$(\$85 - \$5)(1 + 3\%) \approx \$82.4 \tag{18}$$

(ii) Calculate the current value of the forward contract issued three months ago. [15%]

`The value is` $\$12.4/(1 + 3\%) \approx \$12.04$.

(b) On April 28, 2010, 1-year Hellenic Republic bonds were trading at a yield of 24.2% while similar German bonds traded at 0.9%. Assuming that in case of default, the recovery rate for holders of Greek sovereign debt would be 50%, what on April 28 was the market assessment for the probability of default of Greece within 1 year? State the assumptions you make. [30%]

`The implied probability of default over 1 year is roughly` $2 \cdot 23.3\% = 46.6\%$.

(c) Suppose party A has the obligation to deliver to party B a ten-year bond in one year time. To hedge, party A needs to buy the underlying asset and is considering two trading strategies: (i) to fund the underlying bond with a 1 year loan and (ii) to fund the purchase with a stream of overnight loans he would renew on a daily basis.

(i) Which of the two strategies is riskier? [10%]

(ii) Which of the two strategies is more costly on average? [30%]

(*i*): `The strategy of funding with a 1-year loan is riskless, the alternative is risky.`

(*ii*): `On average, assuming absence of arbitrage the two strategy are equally costly on average under the valuation measure since they yield the same asset at expiry. However, the real world interest rate process is such that the yield curve is biased to` be steep for short maturities. Hence, funding with overnight loans is biased to be a less costly strategy.

(e) Consider a swaption contract giving the holder the right to enter into a standard USD swap in 3 years at a fixed rate K. Express the value of the payoff with a formula. [25%]

The payoff has the form $\sum_i \tau Z_T(T_i)(SR_T - K)_+$ where SR_T is the equilibrium swap rate prevailing at time T.

End of Question 5